Absorption cross section of RN black hole

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The behavior of a charged scalar field in the RN black hole space time is studied using WKB approximation. In the present work it is assumed that matter waves can get reflected from the event horizon. Using this effect, the Hawking temperature and the absorption cross section for RN black hole placed in a charged scalar field are calculated. The absorption cross section σ_{abs} is found to be inversely proportional to square of the Hawking temperature of the black hole.

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I. INTRODUCTION

Black holes are natural, stable solutions of Einstein equations in general relativity. A black hole is distinguished by the fact that no information can escape from within the event horizon. The presence of black holes can be inferred only through indirect methods. One of the most useful and efficient ways to study the properties of black hole is by scattering matter waves off them [1]. By studying how a black hole interact with its environment one may understand whether there are ways of making more direct observations of such, in principle, invisible objects. These studies will be useful for understanding of the signals expected to be received by the new generation of gravitational-wave detectors in the near future [2] which is certainly one of the most challenging tasks for modern observational astronomy.

A considerable effort has taken place in studying the waves scattered off by black holes. Both numerical and analytical methods in solving the various wave equations in black hole scattering have been developed [1, 3]. In classical general relativity black hole's properties can be precisely calculated and the black holes may be thought of as astronomical objects with masses about several times of sun. Hawking found when the laws of quantum field theory is applied black holes are not truly black. It possesses entropy and temperature and quantum mechanically a black hole with temperature is able to emit radiation, leading to a situation of particle production int the presence of black holes. When we try to quantize gravitational force, we have to consider how does quantum mechanics affect the behavior of black holes.

Interest in the absorption of quantum waves by black hole was reignited in the 1970s, following Hawking's discovery that black holes can emit, as well as scatter and absorb, radiation [4]. Hawking showed that the evaporation rate is proportional to the total absorption cross section. Unruh [5] found the absorption cross section for massive scalar and Dirac particles scattered off by small non rotating black holes. In a series of papers [6, 7, 8] Sanchez considered the scattering and absorption of massless scalar particles by an uncharged, spherically symmetric black hole.

Another quantum effect of intrest is that event horizons need not be fully absorptive type but can reflect waves falling on it. It is also proposed that event horizons has a finite energy width. 't Hooft [9] explained the horizon of the black hole as a brick wall so that the outer horizon r_+ spreads into a range of $(r_+ - \Delta, r_+ + \Delta)$. Quantum horizon concepts were introduced by Mu-Lin Yan and Hua Bai [10]. The relevant equation governing a scattering process in a black hole space time is analogous to Schrodinger type equations governing scattering phenomena in quantum mechanics. Hence the standard techniques used to study quantum scattering can be used to study scattering problems in black hole space time.

In the present work we study the scattering of charged scalar waves in the Reissner Nordstorm (RN) space-time. Earlier several authors have studied scattering of scalar and Fermi fields under different black hole space time and calculated absorption cross sections. In all these calculations the black hole is assumed to be capable of absorbing the radiation falling on it, but here we consider that both absorption and reflection could take place at the horizon of black holes. Kurchiev [11] has also calculated the absorption coefficient of scalar waves in Schwarzschild space time using the phenomenon of reflection of waves at the event horizon. The absorption coefficient of scalar waves in Schwarzschild de Sitter space time was found earlier [12]. In this work we use WKB approximation, which has been proven to be useful in many cases such as, the evaluation of the quasinormal mode frequencies [13, 14], finding the solution of wave equation in the vicinity of event horizon of black holes, etc. In section II we explain the nature of radial wave functions in different regions of RN space time. Section III contains calculation of absorption cross section for charged scalar wave scattered off by RN black hole, wherein we take into consider both reflection and absorption properties of the black hole horizon. Section IV concludes the paper.

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II. NATURE OF RADIAL WAVE FUNCTIONS IN DIFFERENT REGIONS OF RN SPACE TIME

The metric describing a charged spherical symmetric black hole, written in spherical polar coordinates, is given by

$$ds^2 = \left(1 - \frac{1}{r} + \frac{q^2}{r^2}\right) dt^2 - \frac{1}{\left(1 - \frac{1}{r} + \frac{q^2}{r^2}\right)} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right). \quad (1)$$

The outer and inner horizons of the RN black hole are,

$$r_\pm = \frac{1}{2} \pm \sqrt{\frac{1}{4} - q^2}. \quad (2)$$

The dynamical behavior of a massive charged scalar field under RN background is [15],

$$\Psi_{;ab} g^{ab} + \imath e A_a g^{ab} [2\Psi_{;b} + \imath e A_b \Psi] + \imath e A_{a;b} g^{ab} \Psi + \mu^2 \Psi = 0. \quad (3)$$

The radial part is separated out by putting, $\Psi = \exp(-\imath\epsilon t)\Phi_l(r) Y_{lm}(\theta, \phi)$, where ϵ, l, and m are energy, momentum and its projection, while $\Phi_l(r)$ is radial function, then we will obtain

$$\Delta \Phi_l''(r) + (2r-1)\Phi_l'(r) + \left(\frac{(\epsilon - eA_t)^2 r^4}{\Delta} - \mu^2 r^2 - l(l+1)\right) \Phi_l(r) = 0, \quad (4)$$

where e is the electric charge, $A_t = \frac{q}{r}$ is the electric potential and $\Delta = r^2 - r + q^2 = (r - r_+)(r - r_-)$. Therefore Eq. (4) can be written as

$$\left(r^2 - r + q^2\right) \Phi_l''(r) + (2r-1)\Phi_l'(r) + \left(\frac{(\epsilon - eA_t)^2 r^4}{r^2 - r + q^2} - \mu^2 r^2 - l(l+1)\right) \Phi_l(r) = 0. \quad (5)$$

The radial equation can also be written as

$$\frac{d}{dr}(r^2 - r + q^2)\Phi_l'(r) + \left(\frac{(\epsilon - eA_t)^2 r^2}{\left(1 - \frac{1}{r} + \frac{q^2}{r^2}\right)} - \mu^2 r^2 - l(l+1)\right) \Phi_l(r) = 0. \quad (6)$$

To study the scattering problem we divide the space time into 3 regions [5]. We consider the three different regions starting from the event horizon as shown below.

A. Region 1: Vicinity of horizon: $r \to r_+$

We solve the wave equation near the horizon and also evaluate Hawking temperature using WKB approximation. By using WKB approximation $\Phi = \exp^{-\imath \int k(r) dr}$ in Eq. (5), and equating the real part we will get the radial wave number $k(r, l, \varepsilon)$ from the corresponding equation of motion:

$$k^2(r) = \left(1 - \frac{1}{r} + \frac{q^2}{r^2}\right)^{-1} \left[\frac{(\epsilon - eA_t)^2}{\left(1 - \frac{1}{r} + \frac{q^2}{r^2}\right)} - \frac{l(l+1)}{r^2} - \mu^2\right], \quad (7)$$

which gives,

$$k(r) = \pm \left[(\epsilon - eA_t)^2 r^4 - \left(l(l+1) + \mu^2 r^2\right)\left(r^2 - r + q^2\right)\right]^{\frac{1}{2}} \frac{1}{(r^2 - r + q^2)}. \quad (8)$$

Thus near the outer horizon $r \to r_+$, we will get $k(r \to r_+) = \pm \frac{\xi}{(r - r_+)}$, where $\xi = \frac{(\epsilon - eA_t) r_+^2}{(r_+ - r_-)}$. Therefore the wave function in the region $r \to r_+$ can be written as,

$$\Phi_l(r) \sim \exp(\pm\imath \int \frac{\xi}{(r - r_+)} dr) = \exp\left(\pm\imath\xi \ln(r - r_+)\right), \quad (9)$$

i.e.,

$$\Phi_l(r) \sim \exp\left(\pm\imath\xi \ln(r - r_+)\right). \quad (10)$$

Let us describe its radial motion with the help of the wave function $\Phi(r)$. Using Eq. (9) the wave function in the vicinity of horizon can be written, assuming that the wave gets reflected at the horizon, as

$$\Phi_l(r) \sim \exp\left(-\imath\xi \ln(r - r_+)\right) + \mid R \mid \exp\left(+\imath\xi \ln(r - r_+)\right), \quad (11)$$

where R represents the reflection coefficient and the solution represents the interference between the incident and reflected waves. If $R \neq 0$, there is a definite probability for the incident waves to get reflected at the horizon. The wave function is singular at $r = r_+$. We now consider a point distant $z = r - r_+$, from the horizon and treat z as a complex variable. The above wave function is analytic in z, except for the power type singularity at $z = 0$ which induces a cut emerging from this point on the complex plane z. Let us take r outside the outer horizon region but close to the vicinity of black hole horizon, which means, $0 < z \ll 1$. Now rotate z in the complex z plane over an angle 2π clockwise and examine what happens to the wave function. The validity of semiclassical wave function is justified by keeping $\mid z \mid \ll 1$. This analytical continuation necessarily incorporates a crossing of the cut on the complex plane. Therefore, after finishing this rotation and returning to a real physical value $z > 0$, the wave function acquires a new value on its Riemannian surface. Let it be $\Phi_l^{2\pi}(r)$. Then,

$$\Phi_l^{2\pi}(r) = \rho \exp\left(-\imath\xi \ln(r - r_+)\right) + \frac{\mid R \mid}{\rho} \exp\left(+\imath\xi \ln(r - r_+)\right), \quad (12)$$

where $\rho = \exp(-2\pi\xi)$. The analytically continued function $\Phi_l^{2\pi}(r)$ satisfies the same differential equation as the initial wave function $\Phi_l(r)$. And one has to expect that $\Phi_l^{2\pi}(r)$ must satisfy the same normalization condition as the initial wave function $\Phi_l(r)$. This implies that one of the coefficients, either ρ or $\frac{|R|}{\rho}$ should have an absolute value equal to unity. Since $\rho < 1$, we assume that $\frac{|R|}{\rho} = 1$, thus $R = \exp(-2\pi\xi)$. We see that reflection coefficient is non zero. In other words, black hole horizon is capable of reflection. The probability of reflection from the horizon can be found as $P = \mid R \mid^2 = \exp(-4\pi\xi)$. Since the reflection is taking place against the background of a black hole with temperature T, we see that $P = \exp\left(-\frac{(\epsilon - eA_t)}{T}\right)$, where ϵ is the energy of the particles, e the electric charge and $A_t = \frac{q}{r}$ is the electric potential. Therefore the Hawking temperature of RN black hole is ,

$$T = \frac{r_+ - r_-}{4\pi r_+^2}. \tag{13}$$

B. Region 2: Intermediate region: $r > r_+$

The region is sufficiently away from r_+ but not very far away from r_+ . Here the terms in $(\epsilon - eA_t)^2$ and μ^2 are much smaller than all other terms. Thus we neglect the low energy and momentum in Eq. (5), and for s-wave it becomes,

$$\Phi_0''(r) + \frac{2r-1}{r^2 - r + q^2}\Phi_0'(r) = 0. \tag{14}$$

Therefore $\ln \Phi_0'(r) = -\ln\left(r^2 - r + q^2\right) + \ln C$ and it can be written as,

$$\Phi_0'(r) = \frac{C}{r^2 - r + q^2} = \frac{A}{r - r_+} + \frac{B}{r - r_-}. \tag{15}$$

Since $r > r_+$ we neglect the effect of r_-, and the above equation can be written as,

$$\Phi_0(r) = \int \frac{K}{(r - r_+)\, r} dr, \tag{16}$$

i.e.,

$$\Phi_0(r) = \alpha \ln \frac{(r - r_+)}{r} + \beta. \tag{17}$$

1. *Comparing regions 1 and 2*

The definitions of the above two regions do not really lead to any overlap region. However near the point r_+ one can approximate the solutions by linear combinations of constant terms and terms proportional to $\ln(r - r_+)$. To obtain this consider Eq. (11) which is the wave function in the region $r \to r_+$

$$\Phi_l(r) = \exp(-\imath\xi \ln(r - r_+)) + \mid R \mid \exp(+\imath\xi \ln(r - r_+)). \tag{18}$$

We can take $\exp(\pm\imath\xi \ln(r - r_+)) = 1 \pm (\imath\xi \ln(r - r_+))$, therefore the above equation becomes,

$$\Phi(r) = 1 + \mid R \mid - (1 - \mid R \mid)\, \imath\xi \ln(r - r_+). \tag{19}$$

Eq. (17) can be written as,

$$\Phi_0(r) = \alpha \ln(r - r_+) + \beta. \tag{20}$$

Comparing Eq. (20) with Eq. (19) we get,

$$\alpha = -\imath\xi(1 - \mid R \mid),\ \beta = 1 + \mid R \mid. \tag{21}$$

C. Region 3: Far away from the horizon: $r >> r_+$

Now in the region $r >> r_+$ we can write $2r - 1 = 2r - (r_+ + r_-) = (r - r_+) + (r - r_-)$. Therefore Eq. (5) becomes:

$$\Phi_l''(r) + \left(\frac{1}{r - r_+} + \frac{1}{r - r_-}\right)\Phi_l'(r) \tag{22}$$

$$+ \left(\frac{(\epsilon - eA_t)^2 r^4}{(r^2 - r + q^2)^2} - \frac{\mu^2 r^2}{r^2 - r + q^2} - \frac{l(l+1)}{r^2 - r + q^2}\right)\Phi_l(r) = 0.$$

In the above equation the terms containing energy and mass can be simplified as,

$$\frac{(\epsilon - eA_t)^2 r^4}{(r^2 - r + q^2)^2} = (\epsilon - eA_t)^2 + \frac{2(\epsilon - eA_t)^2 r}{(r^2 - r + q^2)} + \frac{\left(\epsilon - \frac{eq}{r}\right)^2 (r^2 - 2r^2q^2 - q^4)}{(r^2 - r + q^2)^2}, \tag{23}$$

and

$$\frac{\mu^2 r^2}{r^2 - r + q^2} = \mu^2 + \frac{\mu^2 r}{r^2 - r + q^2} - \frac{\mu^2 q^2}{r^2 - r + q^2}. \tag{24}$$

Thus,

$$\frac{(\epsilon - eA_t)^2 r^4}{(r^2 - r + q^2)^2} - \frac{\mu^2 r^2}{r^2 - r + q^2} = (\epsilon - eA_t)^2 - \mu^2 + \frac{\left(2(\epsilon - eA_t)^2 - \mu^2\right) r}{(r^2 - r + q^2)}, \tag{25}$$

$$= p^2 + \frac{\left(p^2 + (\epsilon - eA_t)^2\right) r}{(r^2 - r + q^2)}, \tag{26}$$

where p is the momentum and is given by $p^2 = (\epsilon - eA_t)^2 - \mu^2$. Since r is very large, only terms up

to $\frac{1}{r}$ is taken. Substituting Eq. (26) in Eq. (22) we get,

$$\Phi_l''(r) + \frac{2}{r}\Phi_l'(r) + \left(p^2 + \frac{\left(p^2 + (\epsilon - eA_t)^2\right)}{2r} - \frac{l\,(l+1)}{r^2}\right)\Phi_l(r) = 0. \quad (27)$$

This equation is of Coulomb type where the Coulomb charge is $Z = \frac{(\epsilon - eA_t)^2 + p^2}{2}$ and the solution to this equation can be written,

$$\Phi_l(r) = \frac{1}{r}\left(A_l \exp(\imath z) + B_l \exp(-\imath z)\right), \quad (28)$$

where $z = pr - \frac{l\pi}{2} + \nu \ln 2pr + \delta_t^{(c)}$, where $\delta_t^{(c)} = \arg\Gamma\,(l + 1 - \nu)$, where $\nu = \frac{Z}{p}$ [16]. In the region of large separations $r \gg r_+$. The Coulomb wave function has a regular singularity at $r = 0$ and it has an irregular singularity at $r = \infty$. Let $F_l\,(r)$ be the regular Coulomb wave function and $G_l\,(r)$ be the irregular Coulomb wave function. Then the solution of Coulomb problem can be presented as a linear combination of these two functions:

$$\Phi_l(r) = \frac{1}{r}\left(aF_l\,(r) + bG_l\,(r)\right). \quad (29)$$

In the asymptotic region $r \to \infty$, for Coulomb functions we can use the known formulae, $F_l\,(r) = \sin z$, $G_l\,(r) = \cos z$ where $z = pr - \frac{l\pi}{2} + \nu \ln 2pr + \delta_t^{(c)}$, thus Eq. (29) will be in an asymptotic form:

$$\Phi_l(r) = \frac{1}{r}\left(a \sin z\ + b \cos z\ \right). \quad (30)$$

But we know that, for $l = 0$, [16]

$$F_0\,(r) = cpr\,,\ G_0\,(r) = \frac{1}{c}, \quad (31)$$

where

$$c^2 = \frac{2\pi\nu}{1 - \exp(2\pi\nu)}. \quad (32)$$

Thus Eq. (29) for s wave will be,

$$\Phi_0(r) = acp + \frac{b}{cr}. \quad (33)$$

1. *Comparing Regions 2 and 3*

To compare the wave function in the regions 2 and 3, we write Eq. (17) as,

$$\Phi_0(r) = \alpha \ln\left(1 - \frac{r_+}{r}\right) + \beta \simeq -\frac{\alpha r_+}{r} + \beta, \quad (34)$$

since $\ln\left(1 - \frac{r_+}{r}\right) = -\frac{r_+}{r}$. Thus from Eq. (33) and Eq. (34) we get,

$$a = \frac{1 + \mid R \mid}{pc},\ b = \imath\xi r_+ c\,(1 - \mid R \mid)\,. \quad (35)$$

III. ABSORPTION CROSS SECTION

Now we will find an expression for the absorption cross section of RN black hole. The two terms in Eq. (28) represent the incoming and outgoing waves. The S matrix can be written as the ratio of coefficient of the incoming and outgoing waves. Therefore,

$$S_l = (-1)^{l+1}\frac{A_l}{B_l}\exp(2\imath\delta_l)\,, \quad (36)$$

where A_l represents the amplitude of the incident wave and B_l that of the reflected wave. We have to find S matrix. Since the latter decreases exponentially with energy we will consider first the low energy region $\epsilon \ll 1$, where reflection from the horizon is prominent and restricted to $l = 0$ and denoting s wave as $\Phi_l(r) = \Phi_0\,(r)$. Thus we can find coefficient A_0, B_0 in the latter. Using Eq. (30) we can deduce,

$$A_0 = \frac{a + \imath b}{2\imath},\ B_0 = \frac{-a + \imath b}{2\imath}. \quad (37)$$

Employing Eq. (35) we can find,

$$A_0 = \frac{\left[1 + \mid R \mid - \xi c^2 p\,(1 - \mid R \mid)\, r_+\right]}{2\imath pc}, \quad (38)$$

and

$$B_0 = -\frac{\left[1 + \mid R \mid + \xi c^2 p\,(1 - \mid R \mid)\, r_+\right]}{2\imath pc}. \quad (39)$$

Corresponding S-matrix from Eq. (36) for the s-wave is given by,

$$S_0 = -\frac{A_0}{B_0}\exp(2\imath\delta_0) = \frac{1 + \mid R \mid - \xi c^2 p\,(1 - \mid R \mid)\, r_+}{1 + \mid R \mid + \xi c^2 p\,(1 - \mid R \mid)\, r_+}\exp(2\imath\delta_0)\,, \quad (40)$$

which can be written as,

$$S_0 = \frac{1 - \xi c^2 p r_+ \eta}{1 + \xi c^2 p r_+ \eta}\exp(2\imath\delta_0)\,, \quad (41)$$

where $\eta = \frac{1 - |R|}{1 + |R|}$. The absorption cross section in the low energy limit is given by,

$$\sigma_{abs} = \frac{\pi}{p^2}\left(1 - \mid S_o \mid^2\right) = \frac{\pi}{p^2}\frac{4c^2\xi p r_+ \eta}{\left(1 + \xi c^2 p r_+ \eta\right)^2}. \quad (42)$$

Taking $p = (\epsilon - eA_t)\,v$, we write Eq. (42) as,

$$\sigma_{abs} = \frac{4\pi c^2 r_+^3 \eta}{v\,(r_+ - r_-)\left(1 + \xi c^2 p r_+ \eta\right)^2}. \quad (43)$$

We know that Hawking temperature is given by,

$$T = \frac{r_+ - r_-}{4\pi r_+^2}. \quad (44)$$

Therefore

$$\sigma_{abs} = \frac{c^2 r_+ \eta}{vT\left(1 + \xi c^2 p r_+ \eta\right)^2}. \tag{45}$$

In the expression for absorption cross section we have to substitute for c^2 and η. We know that $\nu = \frac{Z}{p}$, where $Z = \frac{(\epsilon - eA_t)^2 + p^2}{2}$ and therefore $\nu = \frac{(\epsilon - eA_t)^2 + p^2}{2p} = \frac{(\epsilon - eA_t)v}{2}\left(1 + \frac{1}{v^2}\right)$. Hence,

$$c^2 = \frac{\pi\left(\epsilon - eA_t\right) v \left(1 + \frac{1}{v^2}\right)}{1 - \exp\left(-\pi\left(\epsilon - eA_t\right) v \left(1 + \frac{1}{v^2}\right)\right)}, \tag{46}$$

for low energy,

$$c^2 = \frac{\pi\left(\epsilon - eA_t\right) v \left(1 + \frac{1}{v^2}\right)}{1 - \left(1 - \pi\left(\epsilon - eA_t\right) v \left(1 + \frac{1}{v^2}\right)\right)} \simeq 1, \tag{47}$$

and

$$\eta = \frac{1 - \exp\left(-2\pi\xi\right)}{1 + \exp\left(-2\pi\xi\right)} = \tanh \pi\xi \simeq \pi\xi. \tag{48}$$

Substituting Eqs. (47) and (48) in Eq. (45) we find,

$$\sigma_{abs} = \frac{r_+ \pi \xi}{vT\left(1 + \frac{(\epsilon - eA_t)^2 r_+}{16\pi^2 T^2}\right)^2} \simeq \frac{\left(\epsilon - eA_t\right) r_+}{4vT^2}. \tag{49}$$

i.e, the absorption cross section of RN black hole is found to depend inversely on square of Hawking temperature. Now if $q = 0$, the metric becomes of Schwarzschild type. Thus if we substitute $T = \frac{1}{4\pi r}$ and $r_+ = r$ we will get Kuchiev's result [17]

$$\sigma_{abs} = \frac{4\pi^2 r^3 \epsilon}{v}, \tag{50}$$

and in the absence of reflection we arrive at the result obtained by Unruh [5]. Now using Eq. (43) we plot σ_{abs} versus ε. The curve is plotted for RN black hole with reflection and taking charges q=0.1, 0.2, 0.3, 0.4. And we found that absorption cross section decreases when charge is increased from 0.1 to 0.4, i.e., there is more possibility of reflection. It is shown in Figure 1.

In Figure 2 we plot σ_{abs} versus ε for RN black hole with and without reflection and for Schwarzschild black hole with and without reflection. From the plot it is clear that absorption cross section is decreased by the presence of charge. And also found that for RN case the graph is shifted to right.

IV. CONCLUSION

We found the wave function $\Phi_l(r)$ in the vicinity of outer horizon of RN black hole i.e $r \to r_+$ for charged scalar field using WKB approximation. We have also studied the behavior of scattered charged scalar waves in the regions $r > r_+$ and $r \gg r_+$ in low energy limit. By comparing the solutions in the 3 regions viz., $r \to r_+$, $r > r_+$ and $r \gg r_+$, we found the S-matrix and the absorption cross section for RN black hole in the lower energy limit. The absorption cross section is found to be inversely depending on the square of the Hawking temperature. From σ_{abs} of RN, we deduced the absorption cross section of Schwarzschild black hole in the presence of reflection and in the absence of reflection, which agree with the results obtained earlier [5, 17]. By plotting σ_{abs} versus ε plot it is found that absorption cross section is decreased by increasing the of charge in RN black hole.

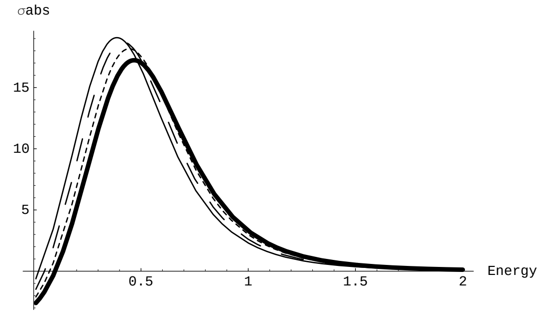

FIG. 1: σ_{abs} versus ϵ for RN black hole with reflection, is plotted for different charges. The solid curve is for $q = 0.1$, dashed curve is for $q = 0.2$, dotted curve is for $q = 0.3$, and bold curve is for $q = 0.4$.

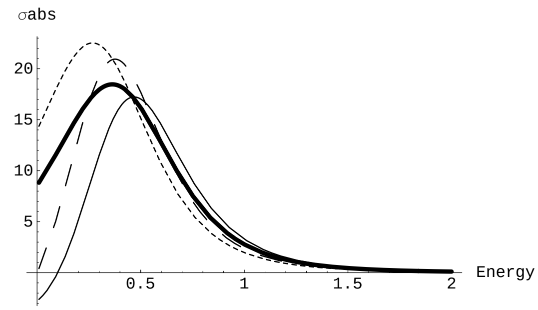

FIG. 2: σ_{abs} versus ε for RN black hole with (solid curve) and without (bold curve) reflection and for Schwarzschild black hole with (dashed curve) and without reflection (dotted curve).

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